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COMMENT

Anderson localization on a $d = 2$ triangular lattice

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Abstract. We show in this article, by using finite size scaling, that the $d = 2$ triangular lattice does not exhibit an insulator–metal transition in constrast to the claim of Srivastava.

The key prediction of the one-parameter scaling theory [1] of the Anderson transition is that a mobility edge exists separating extended from localized states only for $d > 2$. Although some recent models have been formulated which exhibit extended states at certain energies even in 1D [2], the central claims of the scaling theory appear to be intact. However, in a series of recent papers, Srivastava [3] has proposed that quite generally the localization properties of a random lattice are determined more by the underlying connectivity of the lattice rather than by the dimensionality. In particular, he has argued that the $d = 2$ triangular lattice possesses a mobility edge. His results are based on the observation that the upper limit criterion

$$W_c = 4\tilde{K}V \ln(W_c/2V) \tag{1}$$

of Abou-Chacra *et al* [4] for the critical disorder (W_c) beyond which no state is extended, can be used to obtain 'nearly exact' results if an effective connectivity ($\tilde{K}$) is used. Srivastava [3] claims that the appropriate connectivity ($\tilde{K}$) which should be used in conjunction with (1) can be calculated by enumerating the self-avoiding paths on a random lattice that extend to infinity. Implicit in this argument is the assumption that ultimately only such paths contribute to the renormalized perturbation expansion for the self-energy. It is straightforward to verify that (1) has no solution for $\tilde{K} \ll 1.36$. This is particularly fortuitous because the effective connectivity of the simple square lattice is $\tilde{K} = 1.35$. However, because the effective connectivity increases in a complicated way as the number of nearest-neighbours increases, a $d = 2$ lattice can be constructed for which (1) has a non-trivial solution. Srivastava [3] has shown correctly that the triangular lattice possesses an effectivity connectivity of 1.69. Consequently, on his account, the triangular lattice should exhibit an insulator–metal transition when the disorder is less than $W_c = 11.5$. Hence, the triangular lattice provides a general counter-example to the scaling theory of localization (according to Srivastava). Because of the peculiar nature of this result, we have investigated the localization properties of the triangular lattice using the finite size scaling method [5]. From our studies, we show that no insulator–metal transition exists. We conclude, then, that dimensionality is more likely than not the key determining factor in an insulator–metal transition.

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To investigate the localization properties of the triangular lattice, we used transfer matrix methods [5] to calculate the localization length on the standard site-disordered Hamiltonian. The site energies were chosen from a uniform distribution of width W. A constant site-off-diagonal matrix element $V = 1$ was used. We calculated the localization length of a particular eigenstate on triangularly-connected strips of varying widths $M = 5, 9, 13, 17, 21, 25, 29, 33, 37$ and 41, with disorder strengths of $W = 3.5, 4, 4.5, 5, 5.5, 6, 7, 8, 9, 10, 11.5, 13, 14.5, 16, 18$ and 20. The length of the sample was chosen to be $L = 8000$. Although some fluctuations were observed in our numerical simulations for samples of such size, the finite size scaling method still permits a differentiation between localized and extended states. Let $L_c(M)$ represent the localization length for an infinite strip of width M. When a state is exponentially localized, the ratio $L_c(M)/M$ is a decreasing function of M. In fact, $L_c(M)$ will approach some finite value when $M > L_c(M)$. On the other hand, when a state is extended the ratio $L_c(M)/M$ is an increasing function of M. The intermediate or critical case corresponds to $L_c(M)/M$ exhibiting constant behaviour as M increases. Nonetheless, the two extremes are clearly distinguished by plotting $L_c(M)/M$ versus M. The change in the slope of $L_c(M)/M$ beyond some value of the disorder will signify a transition from a metal to an insulator.

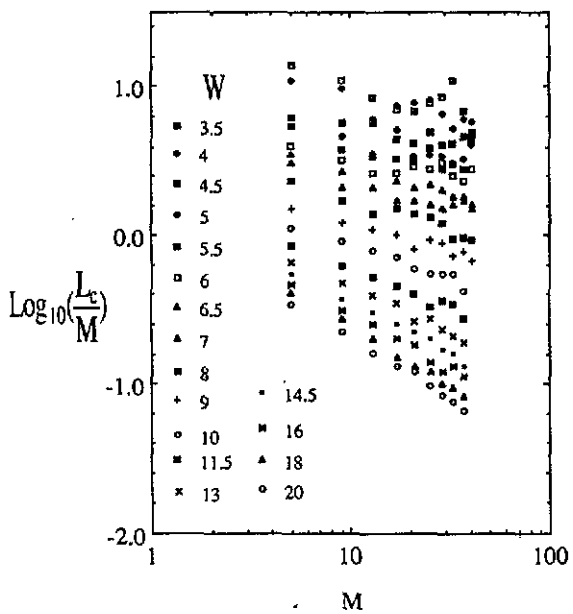

Figure 1. The results of numerical simulations of $L_c(M)/M$ for varying values of the disorder. M is the width of the lattice; $L_c(M)$ is the localization length for the state at energy $E = 2$ on a triangular lattice of length 8000 and width M.

To remove any ambiguity from our results, we first calculated the participation ratio and determined the energy of the most extended state. We found that at $E = 2$, the localization length was largest regardless of the disorder. The deviation of this energy from $E = 0$ (as in the case of the square lattice) is a symptom of the asymmetric nature of the electronic band of the triangular lattice. For the eigenstate

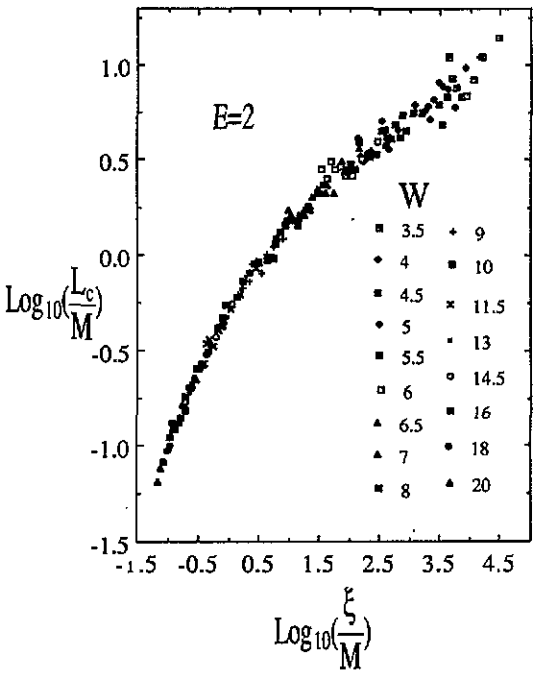

Figure 2. The renormalized localization length $L_c(M)/M$ versus ξ/M, where ξ is the true localization length appropriate for each value of disorder. The universal curve obtained suggests that the triangular lattice obeys one-parameter scaling.

at $E = 2$, we computed $L_c(M)/M$ at the values of M stated above. The raw data are shown in figure 1. As is evident, the slope of $L_c(M)/M$ remains negative even for the smallest value of the disorder $W = 3.5$. At the critical value of $W_c = 11.5$ reported by Srivastava [3], we find no evidence of an insulator–metal transition. In fact, a plot of the participation ratio for $W = 11.5$ reveals that all the states at this energy are well localized. A more transparent way of presenting the numerical data is to scale the results of each simulation by a width-independent localization length ξ that is determined strictly by the disorder. Correct implementation of this procedure results in a universal curve for all of the data, that is $L_c(M)/M = f(\xi/M)$ where f is a universal function. This procedure is certainly warranted if one-parameter scaling is valid. The results of this rescaling are shown in figure 2. As is evident, one-parameter scaling is obeyed in the triangular lattice. Furthermore, because there is only the localized branch, no Anderson transition occurs. As remarked earlier, an insulator–metal transition occurs only if two distinct branches, corresponding to a change of sign of the slope in $L_c(M)/M$, appear as the disorder is varied. Our results support then the scaling theory of localization and not the work of Srivastava.

The failure of the predictions of Srivastava [3] on the triangular lattice led us to test its validity for $d = 3$. For $d = 3$, it is well accepted that the critical disorder for an insulator–metal transition on a cubic lattice is $W_c = 16.5$ [6]. To investigate the value of the connectivity that Srivastava's theory predicts for the cubic lattice, we enumerated the appropriate self-avoiding walks and obtained a value of 1.43. For this value of the connectivity, (1) predicts an insulator–metal transition for $W_c = 7.7$ which

is less than half of the established value of 16.5. We conclude then that Srivastava's approach to the localization problem is not reliable in $d = 3$ and inconsistent with the scaling theory of localization in $d = 2$.

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Reply by V Srivastava

Before coming to the work of Li and Phillips [1] we will make some general observations about the work done so far on localization in two dimensions (2D).

That 2D is somewhat special for localization was noted for the first time by Licciardello and Thouless (LT) [2]. Work in the pre-LT era—whether numerical (including those by Thouless and collaborators) or analytical based on the 'renormalized perturbation expansion (RPE)' using Anderson's original tight-binding formulation [3]—invariably showed a delocalization–localization transition at some critical value of disorder, W_c, in 2D systems [4]. The numerical calculations of LT [2] hinted at the possibility of the localization lengths being very large for $d = 2$ leading them to suggest that the states in 2D systems could be 'neither extended nor exponentially localized'. These ideas led to the scaling theory of localization [5] based on the renormalization-group differential recursion relation following which most people were convinced [6] that all states were localized for $d = 2$ although there remain questions about the single-parameter scaling theory [7], and the problem of the localization transition in 2D for the spin–orbit scattering situation [8] remains unsolved.

In considering numerical calculations there have been problems of varying data interpretation. For instance, Yoshino and Okazaki [9] showed there to be a localization transition in a square lattice but their data were reinterpreted by LT [2] and found to favour their (LT) contention [2]; Stein and Krey [10] initially found evidence for a localization transition in square and triangular lattices but later reanalysed [11] the same data and found support for the scaling picture [5]. Since the numerical works are on finite systems, it is hard to distinguish between the extended states and the

large localized 2D states. This applies equally well to the work mentioned in [4] and that of Li and Phillips [1] and others who calculated the scaling functions†.

The analytical works approach the problem from two opposite directions. The older RPE approaches dealt with the locator perturbation series treating V, the overlap between neighbouring sites, as a perturbation ($V \ll$ disorder, W), whereas in the scaling approach the perturbation theory takes W to be a perturbation ($W \ll V$). The aim of the work in [12] was to examine why the former method unambiguously gave non-zero values for W_c (sometimes rather high) for $d = 2$ as against $W_c = 0$ given by the latter method which is a widely accepted result now—theoretically as well as experimentally. The success was partial—the apparent cause (or one of the causes) for the 'locator expansion' type approaches yielding high values for W_c was found, but the calculations showed complete localization in the honeycomb and square lattices but not in the triangular lattice, thereby revealing an apparent strong dependence of localization on the connectivity constant of the given lattice. The connectivity dependence of localization may well be an artifact of the underlying Cayley tree approach since the only information about a lattice that goes into the calculation is the connectivity of self-avoiding walks of a certain kind in the lattice. Great significance should not be attached to this aspect.

If one agrees with the use of k (explained in [12]) for the connectivity constant of a specially constructed trimmed Cayley tree which corresponds to a given real lattice, and if one has no doubts about the authenticity of the self-consistent theory of Abou-Chacra *et al* [4]. then one should accept the result of [12]. Indeed, there may be certain aspects, yet undiscovered, that may modify the k or even the whole approach. If these can be discovered, the gap mentioned in the previous paragraph could be reduced or removed. Thus the field seems quite open and new ideas within the framework of the locator expansion [3, 4] are called for. That the scaling theory result for W_c in a triangular lattice does not agree with the result of this approach in its present form [12], as shown by Li and Phillips [1] is neither surprising nor unexpected. This has already been discussed at length in [12].

The last part of Li and Phillips [1] paper has direct relevance to the work of [12]. They calculated k (indicated as $\tilde{K}$ in [1]) for the cubic lattice and found $k \approx 1.4$. It is somewhat surprising that k_{cu} (that is, k for a cubic lattice) has been found to be less than k_{tr} (that is, k for a triangular lattice). Note that the coordination number, C, for both these lattices is 6 but K_{cu} (defined in [12]) is larger than K_{tr}, implying that on average a greater number of self-avoiding walk (SAW) trajectories are joined to a site in the cubic lattice than in the triangular lattice. This is a reflection of the fact that the degrees of freedom to move out of a site are greater in the cubic lattice, a 3D lattice, than in the 2D triangular lattice. Similarly, compared with the triangular lattice, the cubic lattice has more trajectories that converge and terminate on a particular site after leaving the same site and making excursions of different lengths. It is the SAW trajectories of the latter kind that interest us in connection with the calculation of k. All the SAWs in a lattice can be divided into two classes : indenfinitely growing or terminating at some stage. We count the latter at different stages—nth step, $(n+1)$th step, etc—and calculate their conectivity, which is k. The argument given above indicates that k_{cu} should result to be greater than k_{tr}.

† Just as Li and Phillips [1] made deductions from $\log_{10}(L_c/M)$ versus M plots, certain approaches mentioned in [4] plot average inverse participation ratio versus disorder for a number of different system sizes to extrapolate the results for the infinite system.

To conclude, more thinking is needed along the lines proposed in [12] to improve the calculation of W_c within the locator expansion framework [3]. The disagreement of the results of [4] and [12] with those of the scaling theory [1, 5, 6] does not reflect that the approaches involved in the former [4, 12] are basically wrong. The work of [12] gives hope to the possibility that the inconsistency between the scaling approach and the locator expansion approches can be removed. Lastly, there seems to be a misunderstanding on the part of Li and Phillips [1] as to what k actually represents.

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